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Can Goal-Free Problems Facilitating Students' Flexible Thinking?

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Abstract. Problem solving is the key of doing and also learning mathematics. It takes also the fundamental role of developing mathematical knowledge. Responding to the current reform movement in mathematics, students are expected to learn to be a flexible thinker. The ability to think flexible is challenged by the globalisation, hence influence mathematics education. A flexible thinking includes ability to apply knowledge in different contexts rather than simply use it in similar context when it is studied. Arguably problem solving activities can contribute to the development of the ability to apply skills to unfamiliar situations. Accordingly, an appropriate classroom instructional strategy must be developed. A cognitive load theory suggests that by reducing extraneous cognitive load during learning could enhance transfer learning. A goal-free problem strategy that is developed based in cognitive load theory have been showed to be effective for transfer learning. This strategy enables students to learn a large numbers of problem solving moves from a mathematics problem. The instruction in a goal-free problem directs students to 'calculate as many solution as you can' rather than to calculate a single given goal. Many experiment research evident goal-free problem enhance learning. This literature review will discuss evidence goal-free problem facilitate students to solve problems flexibly and thus enhance their problem solving skills, including how its implication in the classroom.

INTRODUCTION

Since technology and science increasingly developing, students need to prepared to be a professionals who can adapted and qualified for technological changes. The development flexible thinking is an important goal in mathematics education recently. Flexible thinking skill is also important for real-life and 21st century workforce. Flexible thinking including the adaptability solving problems when condition warrant. Moreover, student require ability to apply knowledge rather than simply retrieve it from its original context. Hence, Flexible thinking is a key skill in learning mathematics. Flexible thinking helping student to find ways and think differently to solve problems and also understanding what and why those steps precisely selected [1].

Students expected use knowledge not only to solve problems within its original context, but also solve problems beyond its original context. Students are expected to recall and use facts, skills, procedures and mathematical ideas to solve problem. Students also need to be able to apply procedures flexibly. Students require to has ability to select appropriate procedure for particular and modify procedure in different condition is educational goals for learning mathematics [2]. In other words, students should be a flexible thinker for solving problems in various contexts.

Problem solving is the main of doing mathematics. Many competencies and skills developed through this, as well as flexible thinking. The ultimate goal of any problem-solving program is to improve students' performance at solving problems correctly [3]. The specific goals of problem-solving in Mathematics are to improve students' self-concepts with respect to the abilities to solve problems; make students aware of the problem-solving strategies; improve

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students' abilities to select appropriate solution strategies; improve students' abilities to implement solution strategies accurately; improve students' abilities to get more correct answers to problems [3]. Problem-based learning can be used to improve students' flexible thinking skills. By enriching and modifying contexts in conventional problem-based learning students would trigger many competencies and skills, including think flexibly.

In this article, we discuss the flexible thinking and goal-free problems interacted. In the first part of this article we present some definitions and theoritical framework of flexible thinking, while the second part led to the conceptual and condition applicability goal-free problems as a instructional design in classroom lead, and the last but not least we proposed implication and summary to covering the discussion.

Flexible Thinking

Fundamental goal of all instruction is to develop skills, knowledge, and abilities that transfer to tasks not explicitly covered in the curriculum [4]. It argued that students should know what and why procedures followed accurately. That flexible thinking skills is one of the important skills for students. The importance of this capability has even become the main goal of school mathematics [5]. Educational goals for the use of mathematical procedures involve flexibility, including the ability to select appropriate procedures for particular problems and modify procedures when conditions warrant [1]. Furthermore, flexible thinking has become a benchmark or indicator of a variety of existing capabilities. Being Able to flexibly solve problems is one of the hallmarks of procedural fluency [1].

A flexible strategy choice includes “the conscious or unconscious selection and use of the most appropriate solution strategy on a given mathematical item or problem, for a given individual, in a given context [6]. Moreover, attribute a broader meaning to flexibility: They define flexibility as knowledge of multiple strategies and the relative efficiency of these strategies [7], flexibility also define as an ability to switch between different strategies [8]. Flexibility referred to the production of some ideas which were used to solve a task [5]. Based on those definitions it is understood that the first most important characteristics in flexible thinking is to have information about multiple strategies. Flexible problem solvers know more than one way to complete tasks. Second, flexibility is required knowledge of strategy efficiency. This means that flexible problem solvers can recognize the which strategies are more efficient than others under particular circumstances.

In this paper, we define flexible ability same as trasnfer ability. Transfer ability define as competencies to use undertsanding from one situastion to another [9]. Since flexible skill define as abilities to apply available knowledge in relatively new situations [10], then we argue that flexible is just the same with transfer ability. One important component of flexibility in mathematics is knowing more than one way to solve a given problem. Flexibility is associated with changing ideas and producing a variety of solutions. [2] Flexibility in problem solving refers to a student’s ability to solve a problem using many different methods or ways. Flexibility in problem posing also refers to a student’s ability to pose or construct problem with divergent solutions [1]. A flexible problem solver must have following abilities: The first is multiple interpretations of data [11]. A flexible problem solver is able to consider several alternative interpretations of a given situation. When the situation warrants a change, the problem solver is able to switch from one interpretation to another. Second is modifying representations. A flexible problem solver chooses an appropriate representation for the task and current situation, for example, between a concrete or abstract representation, a functional or structural representation, or a principle oriented or surface-oriented representation. Third is modification of strategies. A flexible problem solver can change strategies to reflect changes in resources and task demands. These strategy changes might reflect resource usage, or the basic problem solving approach (e.g., from a more goal oriented to a more data-oriented approach, from a top down to a bottom up, from an exploratory to a confirmatory strategy.

Ability to think flexibly consist on several level. There are four criteria were established to determine levels of students’ flexibility: C1. Selection and use of the most appropriate strategy; C2. Changing strategies when it does not work for the solution of a problem (intra task strategy flexibility); C3. Using multiple strategies for the solution of a problem (intra task strategy flexibility); C4. Changing strategies between problems (inter task strategy flexibility) [12]. Flexible thinking skills include the ability to generate ideas, provide answers varied, using a variety of strategies completion, giving examples related to the concept and to find alternative solutions to many different [12]. Based on variety definitions, we conclude criteria of problem to explore flexibility: (a) there are multiple solution strategies, (b) Non-trivial differences exist in qualities of the multiple strategies, (c) choice of strategy can be evaluated as to its appropriateness, and (d) flexibility is relevant whenever it is possible to be strategic.

Goal-Free Problems as An Instructional Classroom Design

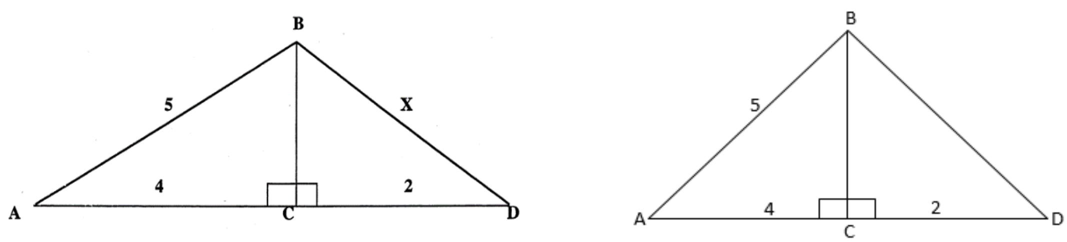
Goal-free effect was the first instructional effect investigated within cognitive load theory. Goal-free problems occur when a conventional problem with a specific goal is replaced by a problems with a non-specific goal [13,14]. Goal-free problems are sometime called no-goal problems. Many researchers study goal-free effect from the domain of geometry. In geometry, goal-free problems indicated by problem statement to ‘calculate the value as many as you can’ instead of ‘caalculate the value of angle ABC’.

Evidence for the effectiveness of goal-free problems is strong. A very simply instructional of goal-free problems evident enhance learning [14], including ability to solve problems in unfamiliar context [15,16]. Goal-free tasks are effective beacause they reduce means-ends problem solving strategies and the extraneous cognitive load assoociated with trying to achieve a specif goal in conventional problem [14]. The various studies describe that a goal-free strategy is effective in transformation problems with a limited problem space that involve only a limited number of possible moves, even with a more extensive problem space [14].

To implement goal-free problems, teacher have to ensure students have enough prior knowledge. Prior knowledge help students to solve problem forwardly. Where prior knowledge or can be called relevant knowledge is absent, the only possibility is to randomly attempt one of the possible moves [13]. In other words, prior knowledge is needed to help students reduce the number of alternative moves that must be randomly tested for solving problems. So, prior knowledge facilitate students to solving problem by the large or even limited number of any possible available moves.

Find the value x of below picture!

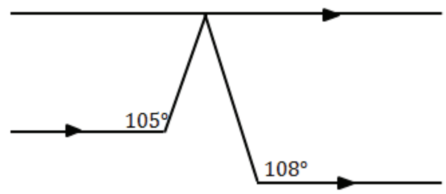
Calculate the unknown segment lenght as many as you can!



(a) (b)

FIGURE 1 (a) is an example of 2-stage Pythagorean Problems which retrieved from Ayres (1998) and **FIGURE (b)** is an example of goal-free problem

Figure 1 (a) shows an example of conventional problem, which indicated a specific goal (to find value x), and Fig.1 (b) shows an example of goal-free problems. Asking students to determine the value of x will occupy their working memory. When students solve a problem, they will work backwards from the goal to the givens using a means-ends strategy [17]. For example (*see figure (a)*), when students are faced a problem requiring them to find value x, they tend to focus on the goal and trying to find a set of connections to the givens. Conventional problem expected students to coherently arrange steps to find out the value of x is not effective for students’ schema acquisition. Unlike conventional problems, no specific-goal in a problem requires students to moves from any possible point. Goal-free problems is not requiring students to work backwards using a means-ends strategy, students are not requiring to solve a goal but students work forwards from the givens. Students could starting to find BC first, then move to another unknown value. To freely work from givens stimulated students to think multiple solution strategies, to think relevant whenever it is possible to be strategic, and evaluate appropriate strategy. Hence, students can flexibly start to calculate the unknown value based on conceptual knowledge.



(c)

FIGURE 2 is a transversal goal-free problem which retrieved from Retnowati (2009)

Figure 2 shows another example of goal-free problems in transversal angles. That problems require students to think relevantly and flexibly to find another unkonwn angles. Students could start calculating angle that relevant into their conceptual knowledge. By managing knowledge to find one by one step forward, students could flexibly apply their knowledge without using a means-ends strategy. Goal-free problems will be hard to solve for students who had no sufficient prior knowledge and conceptual knowledge. Hence, teachers must ensure their students have enough knowledge to solve goal-free problems.

Research has found that goal-free group were superior to conventional group [18]. Several experiments using kinematics and geometry problems with secondary students. The geometry problems used theorems such as vertically opposite angles are equal and the external angles of a triangle equal the sum of the opposite internal angles. Conventional geometry problems required students to find a value for a particular angle in a diagram, whereas goal-free problems asked students to find the values of as many angles as they could. The general procedure was to provide a conventional group with relevant instruction in kinematics or geometry, followed by an acquisition phase involving practice at solving conventional problems. An identical procedure was followed by the goal-free groups except that the practice session used goal-free rather than conventional problems. Common tests using conventional problems were then used to assess learning. Results consistently indicated that the goal-free groups were superior in terms of schema construction. [19]

Ayres (1993) found that on two-step geometry tasks with conventional problems, most errors occurred during the subgoal rather than the goal phase [18]. Working memory load was highest at the subgoal phase, because more elements must be considered at this phase than at the goal phase. In contrast, fewer errors were made by students practicing on goal-free problems with the reduction due to a reduction of working memory load during the non-existing subgoal phase. [19] In the domain of quantitative methods Trumpower, Goldsmith, and Guynn (2004) found that structurally different transfer problems were solved faster after solving problems with nonspecific goals than after solving problems with specific goals [19].

Implications for The Classroom

Goal-free problems occur when a conventional problem with a specific goal is replaced by a problems with a non-specific goal. Teachers can apply this strategy by giving students rich problem to solve as learning activities in the class. A rich problem can facilitate students to think and find as many value as possible. Every students should have sufficient prior knowledge and conceptual knowledge. Thus, teacher should ensure every students have enough knowledge to solve goal-free problems.

Seting students in collaboratively or individually is up to classroom condition. Teacher could ask students solve goal-free problems collaboratively within teams or individually, as long as teacher doesn't provide any assistance. When teachers give some help it means no goal-free problems anymore, because it indicated that problems already has a specific goal. Teachers should allow students to think flexibly without any clue. By that way students lead to ability think flexible and apply their knowledge in different contexts rather than simply use it in similar context when it is studied. After the student has completed, the teacher confirms the answers and concludes the lesson.

Many researchears argued that goal-free problems has clear implication for instruction. According to theoritical framework and a number of conditions, applying goal-free problems can result a very effective alternative to enhance learning outcomes, including ability to think flexibly. Many researchers study goal-free effect from the domain of geometry. In geometry, goal-free problems indicated by problem statement to 'calculate the value as many as you can' instead of 'caalculate the value of angle ABC'.

Research about linear and quadratic equation system indicated that the goal-free problems group more readily switched to a forward-working strategy rather than continuing to use means-ends analysis on subseuent problems. Students in this goal-free group also used equations differently to the goal group. Instead of simply writing down the equations, a feature of the goal group, the goal-free group wrote down the equations and stimultaneously substitute of the given value [20].in

Goal-free problems proven to be a strategy that effectively facilitate students flexible or transfer ability [21]. An experiment in domain transversal angle showed that students learn by goal-free problems strategy indicaed superior in ability ti apply knowledge in different context. Learn by 'calculate thee value as many as you can' is argued could encourage students to be more better in solving unfamiliar context problems.

Goal-free problems indicated enhance ability to apply knowledge in different contexts [21,14,22]. Goal-free problems facilitate students to solve problems beyond the problems they had studied. Altough many researchers study goal-free problem in domain geometry, but goal-free problems also applicable in another domain. Based the researchers result, evidence for the effectiveness of goal-free problems is strong, with the effect obtained under a very wide variety of conditions.

CONCLUSION

According to the literatures review, we suggest goal-free problems strategy is effective for developing flexible thinking in solving mathematics problems. Although a very simple and highly effective mthod to eliminate the negative effect on learning of using search-based problem solving strategies. The basic message of this contribution is goal-free problems can facilitating students to use knowledge and solve problem in unfmailiar contexts. Evidence for the effectiveness of goal-free problems is strong, with the effect obtained under a very wide variety of conditions. We believe there are cogent grounds for instructing novice learners in areas such as mathematics and science to reduce the goalspecificity of problems before solving those problems (e.g., if the goal of a problem is to 'calculate a specific variable', transform this goal into 'calculate the value of as many variables as you can'), and for encouraging instructional designers to consider including goal-free problems in their repertoire of techniques when dealing with those areas in which practice at solving problems is an important instructional procedure.

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